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LQR based tuning of PID controller for magnetic levitation system and its performance comparison with conventional method

Anupama B.[1], Shijoh Vellayikot[2]

PG Scholar, Dept. Of Electronics and Instrumentation, Vimal Jyothi Engineering College, Kannur, India[1]

Research Scholar, Electrical Engineering Department, National Institute of Technology, Calicut, India[2]

Abstract: This paper proposes a Linear Quadratic Regulator (LQR) based tuning of PID controller and is implemented for controlling the ball position of magnetic levitation system. Since the system is highly nonlinear in nature, nonlinear differential equations are used to model the system. For the controller design, this nonlinear model is linearized around the operating point using Taylor series expansion method. Performance of the proposed method is compared with that of PID controller which is tuned using conventional Ziegler Nichol's method. From the simulation studies it is clear that even though both the schemes are capable to control the ball position of magnetic levitation system the proposed method yields better result. Quantitative performance comparison is also made based on the time domain specifications such as settling time, maximum overshoot and steady state error. The result shows that LQR based PID controller outperforms its counterpart as the settling time and maximum overshoot are reduced considerably.

Keywords: Proportional-integral-derivative (PID), Linear Quadratic Regulator (LQR), Magnetic Levitation System, Zeigler Nichol's Method (ZN).

I. INTRODUCTION

The Magnetic Levitation System is a benchmark laboratory model for the understanding of control systems. They are widely used in many engineering systems such as high-speed maglev passenger trains, frictionless bearings, levitation of wind tunnel models, vibration isolation of sensitive machinery, levitation of molten metal in induction furnaces, and levitation of metal slabs during manufacturing.

Magnetic levitation (maglev) technology reduces the physical contact between moving and stationary parts and in turn eliminates the friction problem. Maglev systems are inherently nonlinear, unstable and are described by highly nonlinear differential equations which present additional difficulties in controlling these systems.

The common control approaches to overcome the problem by this system namely linear quadratic regulator (LQR) control and ZN tuning that require a good knowledge of the system and accurate tuning to obtain good performance [1].

Nevertheless, it attributes to difficulty in specifying an accurate mathematical model of the process. This paper presents investigations of performance comparison between conventional (PID) tuning method and linear quadratic regulator (LQR) based PID tuning schemes for a magnetic levitation system.

Here the aim is to stabilize the ball position. Performance of both tuning strategies with respect ball position is examined.

Comparative assessment of both schemes to the system performance is presented and discussed.

II. MODELING OF THE MAGNETIC LEVITATION SYSTEM

Levitation is the stable equilibrium of an object without contact and can be achieved using electric or magnetic forces [3]. In a magnetic levitation, or maglev, system a ferromagnetic object is suspended in air using electromagnetic forces [5,7].

These forces cancel the effect of gravity, effectively levitating the object and achieving stable equilibrium. The system model is given in figure.1

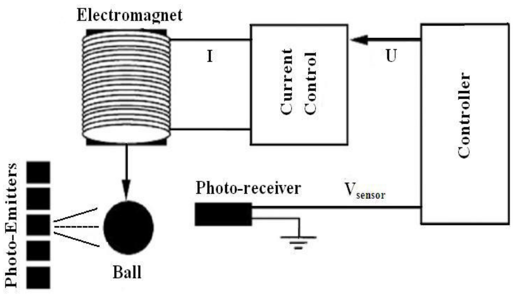

Fig 1. Magnetic levitation system

The maglev system consists of an electromagnet, a steel ball, a ball post, and a ball position sensor. The entire system is encased in a rectangular enclosure which contains three distinct sections. The upper section contains an electromagnet, made of a solenoid coil with a steel core. The middle section consists of a chamber where the ball suspension takes place. One of the electro magnet poles faces the top of a black post upon which a one inch steel ball rests. A photo sensitive sensor embedded in the post measures the ball elevation from the post. The last section of maglev system houses the signal conditioning circuitry needed for light intensity position sensor. The entire system is decomposed into two subsystems, namely, mechanical subsystem and electrical subsystem. The coil current is adjusted to control the ball position in the mechanical system, whereas the coil voltage is varied to control the coil current in an electrical system. Thus, the voltage applied to the electromagnet indirectly controls the ball position. In the following section, we obtain the nonlinear mathematical model of the maglev system and linearize it around the operating region in order to design a stabilizing controller.

Table.1 Magnetic levitation system parameters

Symbol	Description	Value
L_c	Coil inductance	412.5mH
R_c	Coil resistance	10Ω
N_c	Number of turns in the coil wire	2450
l_c	Coil length	0.0825m
r_c	Coil steel core radius	0.008m
R_s	Current sense radius	1Ω
K_m	Electromagnet force constant	6.5308E-005
r_b	Steel ball radius	1.27E-002m
M_b	Steel ball mass	0.068kg
K_b	Ball position sensor sensitivity	2.83E-03m/V
g	Gravitational constant	$9.81 m/s^2$

Applying Kirchhoff's voltage law for the electrical circuit in Figure.2

$$Vc = (Rc + Rs)Ic + L\frac{d}{dt}Ic \tag{1}$$

The transfer function of the circuit can be obtained by applying Laplace transform to Eq. (1)

$$Gc(s) = \frac{Ic(s)}{Vc(s)} = \frac{Kc}{\tau s + 1} \tag{2}$$

Total force experienced by the ball is given by

$$F_c + F_g = -\frac{1}{2}\frac{Km\, Ic^2}{xb^2} + M_b . g \tag{3}$$

Nonlinear equation of motion is given by

$$\frac{d^2xb}{dt^2} = -\frac{1}{2}\frac{Km\, Ic^2}{M_b\, xb^2} + g \tag{4}$$

At the the equilibrium point the derivative terms are set to zero.

$$-\frac{1}{2}\frac{Km\, Ic^2}{M_b\, xb^2} + g = 0 \tag{5}$$

The coil current at the equilibrium point is given by,

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$$Ic0 = \sqrt{\frac{2M_b g}{Km}}\, xb0 \qquad (6)$$

In order to design a linear controller, the system must be linearized around equilibrium point, the point at which the system will converge as time tends to infinity. The nonlinear system equations are linearized around the operating point

$$\frac{d^2xb1}{dt^2} = -\frac{1}{2}\frac{Km\,Ic0^2}{M_b xb0^2} + g + \frac{Km\,Ic0^2 xb1}{M_b xb0^3} - \frac{KmIc\,1Ic0}{M_b xb0^2} \qquad (7)$$

The transfer function will get as

$$G_b(s) = -\frac{Kb\,\omega_b^2}{s^2 - \omega_b^2} \qquad (8)$$

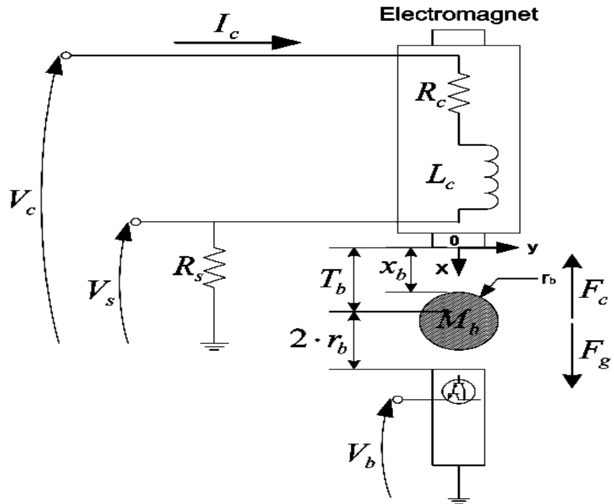

Fig 2. Magnetic levitation schematic diagram

III. LQR BASED OPTIMAL PID TUNING

In this section the gain parameters of PID controller determined using the LQR approach. Here, the points which are important for determining the controller gain alone are explained. In this approach, the error, error rate and integral of error are considered as state variables to obtain the optimal controller gains of the PID controller. The maglev system is represented by the generalised second order transfer function model.

Let the state variables be

$x_1(t) = \int e(t)$ $x_2(t) = e(t)$ and $x_3(t) = \frac{de(t)}{dt}$

from the figure

$$\frac{Y(s)}{U(s)} = \frac{K}{s^2 + 2\zeta^0\omega_n^0 s + (\omega_n^0)^2} = \frac{E(s)}{U(s)} \qquad (9)$$

State space representation of the system is

$$\begin{bmatrix}\dot{x}1\\ \dot{x}2\\ \dot{x}3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ 0 & -(\omega_n^0)^2 & -2\zeta^0\omega_n^0\end{bmatrix}\begin{bmatrix}x1\\ x2\\ x3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ K\end{bmatrix} u \qquad (10)$$

From where the system matrices A and B can be found.

In order to get optimal performance through LQR the quadratic cost function J should be minimised [2,4].

$$J = \int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt \qquad (11)$$

To get J as minimum the controller input u(t) should be optimal

$u(t) = -R^{-1}B^T Px(t) = -Fx(t)$

Where P is the symmetric positive definite solution of the Continuous Algebraic Riccati equation given by

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \qquad (12)$$

The weighting matrix Q is a symmetric positive definite and the weighting factor R is a positive constant. The corresponding state feedback gain matrix is

$$F = R^{-1}B^T P = R^{-1}[0\ 0\ K]\begin{bmatrix}P_{11} & P_{12} & P_{13}\\ P_{21} & P_{22} & P_{23}\\ P_{31} & P_{32} & P_{33}\end{bmatrix}$$

$$= R^{-1}K[P_{31}\ P_{32}\ P_{33}] = -[K_i\ K_p\ K_d] \qquad (13)$$

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The expression for control signal will in the form

$$u(t) = -Fx(t) = -[-K_i - K_p - K_d]\begin{bmatrix} x1(t) \\ x2(t) \\ x3(t) \end{bmatrix} \quad (14)$$

Then we get P in terms of PID controller gains as

$P_{13} = \frac{K_i}{R^{-1}K}\ P_{23} = \frac{K_p}{R^{-1}K}\ P_{33} = \frac{K_d}{R^{-1}K}$

The closed loop system matrix for the system with state feedback gain matrix is

$$A_c = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -R^{-1}K^2P_{13} & (-(\omega_n^0)^2 - R^{-1}K^2P_{23}) & (-2\zeta^0\omega_n^0 - R^{-1}K^2P_{33}) \end{bmatrix} \quad (15)$$

Equating the coefficients of desired and closed loop characteristic equations, the elements of the third row of matrix P is solved.

$$(2\zeta^0\omega_n^0 + R^{-1}K^2P_{33}) = (2+m)\zeta^c\omega_n^c \quad (16)$$
$$(\omega_n^0)^2 + R^{-1}K^2P_{23}) = (\omega_n^c)^2 + 2m(\zeta^c)^2(\omega_n^c)^2 \quad (17)$$
$$R^{-1}K^2P_{13} = m(\zeta^c(\omega_n^c)^3) \quad (18)$$
$$P_{13} = \frac{m(\zeta^c(\omega_n^c)^3)}{R^{-1}K^2} \quad (19)$$
$$P_{23} = \frac{(\omega_n^c)^2 + 2m(\zeta^c)^2(\omega_n^c)^2 - (\omega_n^0)^2}{R^{-1}K^2} \quad (20)$$
$$P_{33} = \frac{(2+m)\zeta^c\omega_n^c - 2\zeta^0\omega_n^0}{R^{-1}K^2} \quad (21)$$

The remaining unknown elements of 'P' matrix can be determined by solving the algebraic Riccati Equation. With the known third row elements of P matrix the other elements of P and Q matrices can be obtained as

$$P_{11} = (\omega_n^0)^2P_{13} + R^{-1}K^2P_{13}P_{23} \quad (22)$$
$$P_{12} = 2\zeta^0\omega_n^0P_{13} + R^{-1}K^2P_{13}P_{23} \quad (23)$$
$$P_{13} = 2\zeta^0\omega_n^0P_{23} + R^{-1}K^2P_{23}P_{33} + (\omega_n^0)^2P_{33} - P_{13} \quad (24)$$
$$Q_1 = R^{-1}K^2P_{13}^2 \quad (25)$$
$$Q_2 = R^{-1}K^2P_{23}^2 - 2(P_{12} - (\omega_n^0)^2P_{23}) \quad (26)$$
$$Q_3 = R^{-1}K^2P_{33}^2 - 2(P_{23} - 2\zeta^0\omega_n^0P_{33}) \quad (27)$$

IV. PERFORMANCE COMPARISON OF LQR WITH CONVENTIONAL TUNING

In this section the proportional, integral and derivative gains of PID controller is find out using conventional tuning algorithm ie, Zeigler Nichol's open loop tuning method. This method of finding controller setting was developed by Zeigler and Nichols also referred as process reaction method [6]. This method can be used for only systems with self regulation. In a typical open loop controller response, the disturbances applied at two times. We expressed the deviation as the percentage range. Using these time constant and lag time is measured [9].

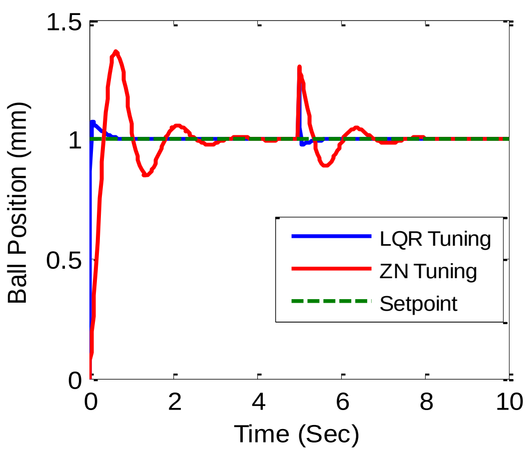

Fig. 3 PID controller response using LQR and ZN tuning methods

The simulation studies are carried out in SIMULINK platform by taking the open loop parameters of magnetic levitation system[8] as K=7, ω=1 .8 and ζ =0.8. From fig.3 it is clear that both tuning schemes are suited for obtaining controller gains and hence to attain the control strategy. Comparing all characteristics from Table.2 LQR tuning yields

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better result by a fast settling time and reduced disturbance rejection time. And also the maximum overshoot is reduced considerably. However, for the overall performance by considering both ball positions, both tuning methods are successfully designed and in fact LQR controller has the best response and better performance which satisfy the design criteria very much.

Table.2 Time response specifications

Time response specifications	LQR	ZN
Settling Time (Sec)	0 .75	3.95
Maximum Overshoot (%)	7	36
Disturbance Rejection Time (Sec)	0.9	3.2

V. CONCLUSION

In this paper, two tuning methods such as LQR and PID are successfully designed. Based on the results and the analysis, a conclusion has been made that both of the tuning method linear quadratic regulator (LQR) and conventional tuning (ZN) are capable of tracking the ball position of the linearized system. All the successfully designed methods were compared. The responses of each tuning were plotted and are summarized in Table 2. Simulation results show that LQR controller has better performance compared to ZN open loop method of maglev system. Further improvement need to be done for both of the tuning.

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BIOGRAPHIES

Anupama B. is currently an M. Tech scholar in Control and Instrumentation, Kannur University, Kerala, India. She received her B. Tech degree in Applied Electronics and Instrumentation from Calicut University. Her area of interests includes control system design, System identification, Process control and Neural Network based controllers. Ms. Anupama is a member in IRED.

Shijoh V. is a Research scholar in the Department of Electrical Engineering, National Institute of Technology, Calicut, India. He received B. Tech in Electrical and Electronics Engineering from Calicut University, Kerala, India and M. E. in Instrumentation Engineering from MIT Campus, Anna University, and Chennai, India. His research interests include Process modelling and control, State estimation, and advanced model based control strategies. He has published papers in national and international refereed journals and conferences. Mr. Shijoh V. is a member in the IRED, IACSIT and IAENG.